Physics Qualifying Examination

September 20, 1988
2:00 p.m. PSTR 105

MATHEMATICAL PHYSICS

(One Reference Book of your choice. No notes)
Write Problems 1 & 2 in one Blue Book and 3 - 5 in another Blue Book

1. A drum head is stretched over a square frame of side L. Oscillations of the drum satisfy a wave equation

$$\nabla^2 Z(x,y,t) - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} Z(x,y,t) = f(x,y,t)$$

with boundary conditions $Z = 0$ on the boundaries. Assuming that the driving term is of the form

$$f(x,y,t) = g(x)h(y)\delta(t)$$

find a series expansion for the solution $Z(x,y,t)$.

2. (a) The imaginary part of the frequency dependent susceptibility for a system is given by

$$\mathrm{Im}\,\chi = \frac{\omega\Gamma}{\omega^2+\Gamma^2} \quad .$$

Find the real part of the susceptibility.

(b) Solve the integral $\int_0^{2\pi} \frac{\theta\, d\theta}{2+\cos\theta}$.

3. Find the Green's function for the solution $y(x)$ of the inhomogeneous differential equation

$$y'' + y' - 2y = g(x)$$

on the interval $0 \le x < \infty$, subject to the boundary condition $y(0) = y(\infty) = 0$.

4. Solve the integral equation

$$f(x) = g(x) + \lambda \int_{-\infty}^{\infty} g(x-y) f(y) dy$$

for arbitrary value of λ, where

$$g(x) = 0 \text{ for } x < 0$$
$$= e^{-x} \text{ for } x > 0$$

and where $f(x)$ is required to remain finite as $x \to \pm \infty$.

5. A set of orthogonal polynomials, $H_n(x)$, may be defined by the generating function

$$\sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!} = e^{2tx-t^2}$$

Evaluate the integral

$$\int_{-\infty}^{\infty} e^{-2x^2} [H_n(x)]^2 dx \quad .$$